Crude Oil Prices: Trends and Forecasts

by

Hossein Askari and Noureddine Krichene

Abstract

Following record low interest rates and a rapidly depreciating US dollar, crude oil prices have come under increasing pressure and seem boundless. Oil price process parameters changed drastically during 2003M5–2007M10 toward consistently rising prices. Short-term forecasting would imply the persistence of observed trends, as market fundamentals and underlying monetary policy have been supportive of these trends. Market expectations derived from option prices anticipated further surge in oil prices and indicated a significant probability of right tail events. Given the combination of explosive trends in the price of other commodities, a depreciating US dollar, and weakening financial conditions, recent trends in oil prices might not persist without triggering a world wide economic recession and regressive oil supply as oil producers become wary about inflation. Restoring stable oil markets, through tightening of monetary policy, is essential for durable growth and price stability.

JEL Classification Numbers: C22, E31, E52, Q41, Q43.
Keywords: Characteristic function, crude oil prices, density forecast, Esscher transform, Fourier transform, inverse problem, normal inverse Gaussian, monetary policy, option prices.
Authors' E-Mail Address: HGAskari@AOL.com;
nkrichene@imf.org - nnkrichene@hotmail.com

Authors' affiliations:
Hossein Askari is Iran Professor of International Business and International Affairs at the George Washington University and Noureddine Krichene is Economist at the International Monetary Fund. The views expressed in this paper are those of the authors.

I. Introduction

By maintaining upward persistence since early 2003 and reaching about US$100/barrel, crude oil prices have remained under intense pressure and have seemed boundless; the rapid increase in oil prices by causing other prices to rise may broaden inflationary pressures and decelerate world economic growth. All the more worrisome, recent upsurges in oil prices were taking place in midst of rising trends in general commodity prices, instability in housing, equity, and credit markets, and a depreciating dollar. The fast rise in commodity prices, including oil, could be seen as delayed effect of excessively expansionary monetary policies during 2001–04 when key interest rates were forced down to post WWII record levels. Such monetary expansion led to high world economic growth and consequently higher world demand for oil and non-oil commodities. Supply of crude oil and other commodities being rigid, or increasing at a slower pace than demand, resulted in the highest level of inflation for commodities markets in the post WWII era. With real interest rates turning low or negative, pressure on real aggregate demand, and therefore on oil markets, might not subside. Relaxation of monetary policy in August─December 2007 immediately set off new spiral in commodities price inflation and currency depreciation.[1]

In this paper we analyze oil price developments during 2000M1–2007M10. To exhibit the effect of monetary policy, we examine two sub-samples 2000M1–2003M4 and 2003M5–2007M10. We assume oil prices are driven by Levy processes (LP) of generalized hyperbolic (GH) type and use daily data to estimate the parameters of these processes. Combining features of normal and stable distributions and offering more flexibility than Poisson-type processes, which were known to model finite large jumps, GH distributions have gained wide popularity in modeling stock market indices. In view of their success in modeling financial time-series, many authors have advocated Levy processes of hyperbolic type. In this respect, the hyperbolic distribution was proposed for modeling LP by Barndorff-Nielsen (1977), Barndorff-Nielsen and Blaesild (1983), Bibby and Sørensen (1997 and 2003), Eberlein and Keller (1995), and Prause (1999). Barndorff-Nielsen (1995), and Rydberg (1997) proposed the normal inverse Gaussian (NIG) distribution; and Madan et al. (1998) applied the variance gamma distribution. GH processes have become appealing for their ability to account for salient features of high frequency financial time-series, namely asymmetry, frequent small and large jumps, and to reduce the smile in option prices (Eberlein et al. 1998).

Inability of Gaussian processes to fit high frequency financial data was underscored by Fama (1965) and Mandelbrot (1963); both authors proposed stable distributions for modeling

---

[1] In August─December 2007, some key discount rates were cut, large amounts of liquidity were injected, and successive cuts in the federal funds rate were undertaken. Indeed, as cuts in interest rates were priced in before their announcements, pressure on oil prices increased prior to actual implementation of these cuts.

skewness and kurtosis; however, stable distributions did not have finite variance and therefore were not appealing for modeling financial time-series. Besides the ability to account for skewness and kurtosis, GH distributions remedy the shortcomings noted in Black-Scholes model (1973) with respect to implausibility of the normality assumption and constancy of the variance of the distribution. To the extent that GH distributions were constructed as mixtures of variance-mean normal distribution with time varying stochastic variance, or equivalently, as Brownian motion subordinated to increasing positive stochastic processes, they can account for stochastic volatility and allow the attenuation of the smile in short-term options (Carr et al. 2003). As a precursor to application of subordinated process for fitting financial time-series Clark (1973) introduced Bochner's concept of a subordinate stochastic process as a model for speculative price series. Clark showed that the concept of subordination would allow the use of finite variance distribution to obtain a mixture distribution, where the mixing distribution is an increasing positive Levy process that has finite moments. He showed, with both discrete Bayes' test and Kolmogorov-Smirnov test, that finite-variance distributions subordinate to the normal, namely a lognormal-normal distribution, fit cotton futures price data better than members of the stable family.

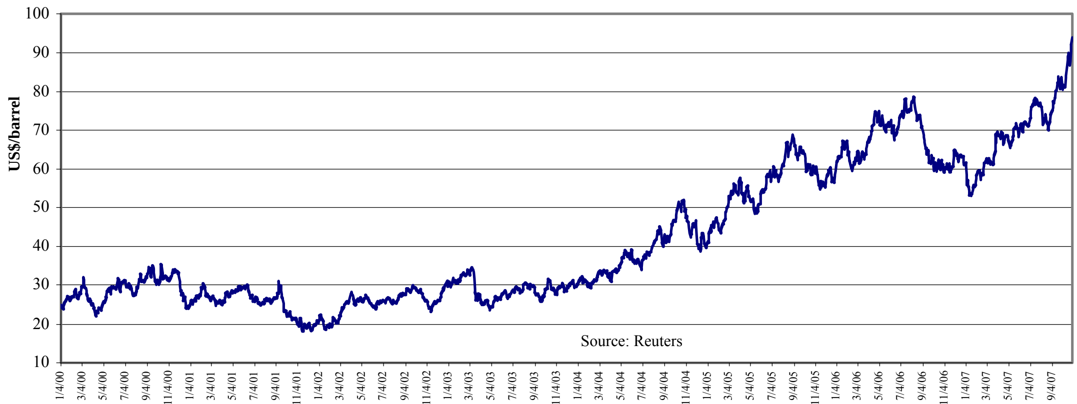
In this paper, we demonstrate that Normal Inverse Gaussian (NIG) process closely fits oil price returns during 2000M1–2003M4 and 2003M5–2007M10; parameters of the process have, however, changed: mean return increased due to persistence in upward trend, and kurtosis has declined due to higher predictability in oil prices. Estimated parameters of the NIG process are in conformity with findings for the empirical distribution of oil price returns. To be applicable for pricing derivatives, statistical distributions had to be adjusted for market price risk and turned into martingale processes. This is done through applying Esscher transform to the statistical process. Besides estimating oil price returns distribution from time-series data, oil price returns distribution was also estimated from cross-section data from call and put option prices on November 2, 2007 for end-December 2007. Implied-risk neutral distribution based on NIG shows that traders were expecting further rise in oil prices and were assigning significant probabilities for right-tail events.

The paper is structured as follows: in Section II we study daily oil prices during 2000M1–2007M10; in Section III we present the theoretical framework for Levy processes; in Section IV we present the normal inverse Gaussian (NIG) distribution; in Section V we present the statistical estimation of the NIG distribution based on daily oil price data for 2000M1–2003M4 and 2003M5–2007M10; in Section VI we derive the risk-neutral distributions from statistical LP by applying Esscher transform to these processes; in Section VII we derive density forecast from oil options prices; and our conclusions follow in Section VIII.

II. Recent Evolution of Oil Prices

Looking at daily data for light crude futures prices from January 4, 2000 to October 29, 2007 (Figure 1), the behavior of oil prices showed two distinct patterns: relative stability during 2000M1–2003M4, around a mean of US$27 per barrel, and strong upward deterministic trend and persistence during 2003M5–2007M10, with prices rising progressively to cross US$96/barrel mark in October 2007, and showing no sign of stability around a mean.[2] The upward trend became predictable and was the longest upward trend in post-war oil prices. Past upward trends lived on average two to three years, while the present one has so far spanned more than four years.

Figure 1. Oil Daily Futures Prices, January 2000ŠOctober 2007



The structure of the oil price process has changed considerably as conveyed by the two regressions below, namely in the second sub-period the lag structure was shorter and a deterministic trend became significant (we denote oil prices by S_t and apply an autoregressive moving average (ARMA) representation):

<u>Sub-period: 2000M1–2003M4</u>

S = 1.325*S(-1) - 1.195*S(-2) + 0.842*S(-3) – 0.191*MA(1)+0.778*MA(2)+0.214*MA(3)+0.769 (1)
(t=23.1) (t=-13.7) (t=14.8) (t=-3.0) (t=13.1) (t=6.1) (t=2.7)

R^2=0.97; DW=2.01

<u>Sub-period : 2003M5–2007M10</u>

S = 0.990*S(-1) + 0.307 + 0.000537*TREND+0.098*MA(1) (2)
(t=207.9) (t=2.23) (t=2.26) (t=3.31)

R^2=0.99; DW=2.00

The empirical distribution of oil price returns (Figure 2a and 2b), defined as $x_t = \log S_t - \log S_{t-1}$, showed considerable change in moments between the two sub-periods

[2] Light crude futures prices from Reuters, 1986 observations.

(Table 1).[3] Mean return was -0.005 in the first sub-period, equivalent to a decline in oil prices of 1.28 (=-0.005*255) percent per year; it rose to 0.12 percent in the second sub-period, equivalent to an increase in oil prices of 30.6 (=0.12*255) percent per year. Volatility was high for daily data. Although falling from 2.226 (annualized to 35.6(=2.226* $\sqrt{255}$) percent) to 1.686 (annualized to 26.9(=1.686* $\sqrt{255}$)) percent), volatility was high, indicating that oil markets were sensitive to news and small shocks and conducive to speculation. Skewness turned out to be small in both sample periods, indicating that the distributions of returns were symmetric. Kurtosis declined to 3.87 from 4.95. Normality assumption for price returns was rejected for both sample periods, indicating essentially pre-eminence of large jumps in daily oil prices.

Table 1. Descriptive Statistics for Oil Price Returns

2000M1–2003M4		2003M5–2007M10	
Mean	-0.005312	Mean	0.118992
Median	0.000000	Median	0.128184
Maximum	11.23630	Maximum	7.356257
Minimum	-9.436276	Minimum	-6.080348
Std. Dev.	2.225607	Std. Dev.	1.686031
Skewness	0.056786	Skewness	-0.021088
Kurtosis	4.954168	Kurtosis	3.873152
Jarque-Bera	133.9488	Jarque-Bera	36.55292
Probability	0.000000	Probability	0.000000

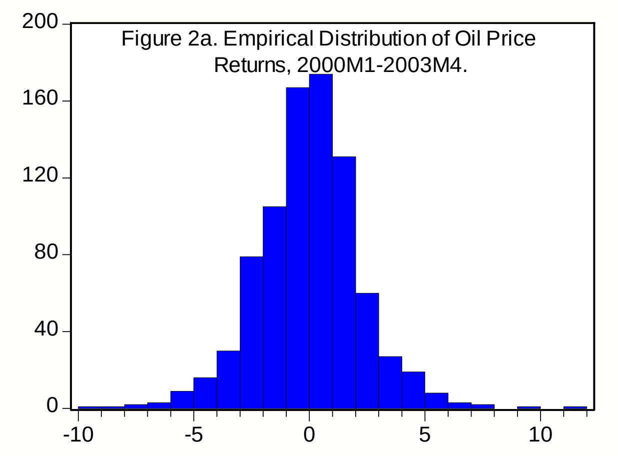


Figure 2a. Empirical Distribution of Oil Price Returns, 2000M1-2003M4.

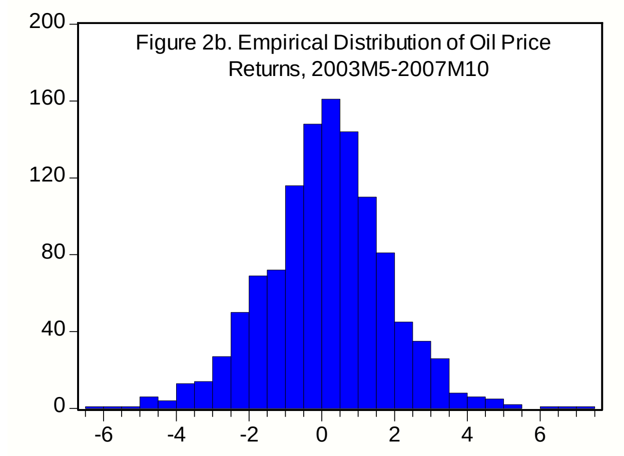


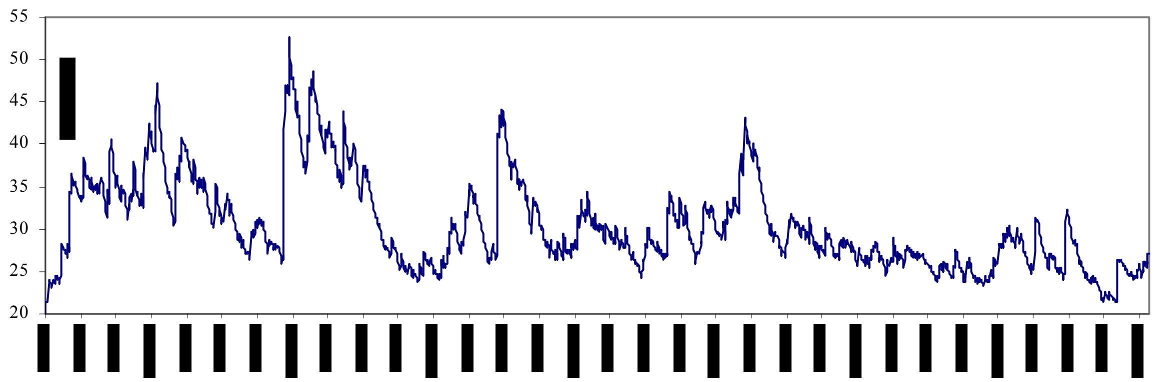
Figure 2b. Empirical Distribution of Oil Price Returns, 2003M5-2007M10

[3] Skewness and kurtosis are defined as $Skewness = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{y_i - \bar{y}}{\hat{\sigma}}\right)^3$ and $Kurtosis = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{y_i - \bar{y}}{\hat{\sigma}}\right)^4$, respectively.

These empirical parameters showed that oil markets were out-of-equilibrium in the second sub-period and were driven by a strong upward trend reflecting rapidly growing demand for oil, rigidities in oil supply, and rising tensions in oil prices.

Volatility, measuring risk and uncertainty, is a key variable in financial markets. High volatility would increase the volume of trade in derivatives markets, both in speculative and hedging activities. Volatility, estimated using GARCH(1,1)[4] (Figure 3), appeared high for daily data and exhibited periods of volatility clustering, followed by periods of mean reversion. When annualized, volatility was high for the first sub-period at 35 percent. In line with the findings of the ARMA model, volatility fell sharply to 22 percent for the second period indicating a solid deterministic trend in oil prices.

Figure 3. Oil Price Returns GARCH(1.1) Volatility, January 2000-October 2007



III. Modeling Oil Prices as Levy Process

Levy processes offer flexibility for accounting for basic features of financial series, namely skewness, excess kurtosis, and frequent small and large jumps.[5] The stochastic differential equation (SDE) underlying asset price is:[6]

---

[4] GARCH stands for Generalized Autoregressive Conditional Heteroskedasticity.

[5] A process $X = (X_t)_{t \geq 0}$ with $X_0 = 0$ is a Lévy process if it possesses the following properties: (i) independent increments: for every increasing sequence of times $t_0, \ldots, t_n$, the random variables $X_{t_0}, X_{t_1} - X_{t_0}, \ldots\ldots, X_{t_n} - X_{t_{n-1}}$ are independent, (ii) stationary increments: the law of $X_{t+h} - X_t$ does not depend on t, and (iii) stochastic continuity: $\forall \varepsilon > 0$, $\lim_{h \to 0} P(|X_{t+h} - X_t| \geq \varepsilon) = 0$. i.e., discontinuity occurs at random times. Levy processes are limits of random walks and are infinitely divisible into independent and identically distributed (i.i.d.) random variables.

$$dS_t = \rho S_{t-}dt + \sigma S_{t-}dZ_t \quad (3)$$

S_t denotes oil price, S_{t-} denotes limit from left, Z_t is a LP, and ρ and σ are drift and volatility parameters.[7] Solution of this equation is the well-known Doléans-Dade (or stochastic) exponential given by:

$$S_t = S_0 \exp\left(\rho t + \sigma Z_t\right)\prod_{\tau \le t}\left(1+\sigma\Delta Z_\tau\right)\exp\left(-\sigma\Delta Z_\tau\right) \quad (4)$$

Here $\Delta Z_\tau = Z_\tau - Z_{\tau-}$ denotes jump at time τ. Clearly, the condition $\left(1+\sigma\Delta Z_\tau\right) \ge 0$ has to hold for S_t to be nonnegative, i.e., $S_t \ge 0$. Thus, jumps are bounded below by $\Delta Z_\tau \ge -\frac{1}{\sigma}$. The solution (4) does not enable easy manipulation of the price process such as computation of compounded returns or simulation of the price path. To obtain an easier form for LP, the SDE is reformulated as: [8]

$$dS_t = \rho S_{t-}dt + \sigma S_{t-}dZ_t + S_{t-}\left(e^{\sigma\Delta Z_t} - 1 - \sigma\Delta Z_t\right) = \rho S_{t-}dt + \sigma S_{t-}\left(dZ_t + e^{\sigma\Delta Z_t} - 1 - \sigma\Delta Z_t\right)(5)$$

Similar to a Poisson process, jumps now appear explicitly in the dynamics of the oil process. This representation of SDE arises from an application of Ito's formula, and yields a simpler solution for the price process:

$$S_t = S_0 \exp\left(\rho t + \sigma Z_t\right) \quad (6)$$

IV. Oil Price Process as Normal Inverse Gaussian Process

In this section, oil price returns, defined as $x_t = \log S_t - \log S_{t-1} = X_t - X_{t-1}$, are analyzed as a normal inverse Gaussian (NIG) distribution, where $S_t = S_0 \exp\left(X_t\right)$. NIG distribution is a special case of the generalized hyperbolic distribution $\left(GH\right)$. The GH distribution was essentially due to Barndorff-Nielsen (1977). Let X be a normal distribution, i.e. $X \sim N\left(\mu + \beta z, z\right)$ and let Z be a generalized inverse Gaussian (GIG) law, i.e. $Z \sim GIG\left(\lambda, \delta, \gamma\right)$. Define $X = \mu + \beta Z + V\sqrt{Z}$, where $V \sim N\left(0,1\right)$, then X is said to have a

---

[6] This is analogous to Black-Scholes (BS) model where asset prices were given $dS_t = \rho S_t dt + \sigma S_t dW_t$, $t \ge 0$, S_t is price at time t, W_t is a Wiener process with $dW_t \sim N\left(0, dt\right)$, ρ and σ are drift and volatility, respectively. By Ito Lemma, the solution is: $S_t = S_0 e^{(\rho - \frac{1}{2}\sigma^2)t + \sigma W_t}$.

[7] The SDE can also be written as: $dS_t = \rho S_{t-}dt + \sigma S_{t-}dZ_t = S_{t-}dX_t$ where $X_t = \rho t + \sigma Z_t$.

[8] If the SDE is formulated as: $dS_t = S_{t-}dX_t$, the reformulated SDE will be: $dS_t = S_{t^-}\left(dX_t + e^{\Delta X_t} - 1 - \Delta X_t\right)$.

GH distribution obtained as a normal variance-mean mixture where the mixing distribution is $Z \sim GIG(\lambda,\delta,\gamma)$; or equivalently, X is obtained as a normal distribution subordinated to a GIG process. It is written as:

$$f_{GH}(x;\lambda,\alpha,\beta,\delta,\mu)=\int_0^{\infty} f_{Normal}(x;\mu+\beta z,z)\,f_{GIG}(z;\lambda,\delta,\gamma)dz \quad (7)$$

Let $\gamma=\sqrt{\alpha^2-\beta^2}$, the parameters of the GH distribution are μ , δ , α , β , and λ , measuring respectively location (μ), scale (δ), steepness of tail (α), skewness of distribution (β), and distribution class (λ). The parameters must satisfy: $\lambda \in R$, $\mu \in R$, $\delta \geq 0$, and $0 \leq |\beta| < \alpha$. GH distribution has more parameters than either stable or normal distribution and provides therefore more flexibility for controlling skewness and tail thickness of the distribution. Parameters α and β are also called shape parameters; higher value for α indicates steeper tail, and β =0 indicates symmetric distribution. Different parameterizations for distribution shape are proposed (see Prause, 1999) and are:

First parameterization: α , β , δ , μ ; 2nd parameterization: $\zeta=\delta\sqrt{\alpha^2-\beta^2}$, $\rho=\dfrac{\beta}{\alpha}$, δ , μ ;

3rd parameterization: $\xi=(1+\zeta)^{-\frac{1}{2}}$, $\chi=\xi\rho$, δ , μ ; and 4th parameterization: $\bar{\alpha}=\alpha\delta$, $\bar{\beta}=\beta\delta$, δ , μ .

NIG distribution was introduced by Barndorff–Nielsen (1995) as a subclass of GH laws obtained for $\lambda=-\dfrac{1}{2}$.[9] The density function for NIG is:

$$nig\,(x;\alpha,\beta,\delta,\mu)=\frac{\delta\alpha}{\pi}\exp\left(\delta\sqrt{\alpha^2-\beta^2}+\beta(x-\mu)\right)\frac{K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\sqrt{\delta^2+(x-\mu)^2}} \quad (8)$$

where: $x,\mu \in R, 0 \leq \delta, 0 \leq |\beta| \leq \alpha$,and K_1 is modified Bessel function of third kind with index 1.[10] Let $\gamma=\sqrt{\alpha^2-\beta^2}$, moments of NIG are:

---

[9] Thus, $GH(\alpha,\beta,\mu,\delta,\lambda=-1/2)=NIG(\alpha,\beta,\mu,\delta)$. NIG process can be related to a Brownian motion time-changed by an Inverse Gaussian process (IG). Let $W=\{W_t, t\geq 0\}$ be a Brownian motion and let $IG=\{IG_t, t\geq 0\}$ be an IG process with parameters $a=1$ and $b=\delta\sqrt{\alpha^2-\beta^2}$, with $\alpha>0$, $-\alpha<\beta<\alpha$ and $\delta>0$; then the process: $X_t=\beta\delta^2.IG_t+\delta W\left(IG_t\right)$ is an $NIG(\alpha,\beta,\delta)$ process with parameters α,β,δ . An equivalent parameterization of NIG process is a Brownian motion with drift θ and volatility σ , $B(t)=\theta t+\sigma W(t)$, computed at random time given by a gamma process $G(1,\upsilon)$: $X_{NIG}(t;\sigma,\upsilon,\theta)=\theta.G_t^{\upsilon}+\sigma W(G_t^{\upsilon})$.

$$E[X] = \mu + \delta\left(\frac{\beta}{\gamma}\right) \quad (9)$$

$$Var[X] = \delta\left(\frac{\alpha^2}{\gamma^3}\right) \quad (10)$$

$$Skew[X] = 3\left(\frac{\beta}{\alpha}\right)\left(\frac{1}{(\delta\gamma)^{1/2}}\right) \quad (11)$$

$$Kurt[X] = 3\left(1 + 4\left(\frac{\beta}{\alpha}\right)^2\right)\left(\frac{1}{(\delta\gamma)}\right) \quad (12)$$

The moment generating function (MGF) of NIG is:

$$M_{NIG}(u) = e^{\mu u}\,\frac{e^{\delta\sqrt{\alpha^2-\beta^2}}}{e^{\delta\sqrt{\alpha^2-(\beta+u)^2}}} = e^{\mu u+\delta\left(\sqrt{\alpha^2-\beta^2}-\sqrt{\alpha^2-(\beta+u)^2}\right)} \quad (13)$$

satisfying $|\beta + u| < \alpha$. Simulation of NIG for forecasting purposes was studied by Rydberg (1997) based on algorithms developed by Dagpunar (1989).

NIG motion, as well as other subclasses of hyperbolic distributions, is LP that has the property of being pure jump and infinite activity models. Its representation as a time-changed Brownian motion allows the modeling of time change which itself reflects the intensity of economic activity through news arrival and trades. Tractability of the characteristic function (CF) of the NIG and other subclasses of the GH allows to recover option prices through fast Fourier transform (FFT). Knowledge of CF enables to recover the probability distribution through numerical inversion (Davies, 1973) as:

$$F(x) = \frac{1}{2} + \frac{1}{2\pi}\int_0^\infty \frac{e^{iux}\phi(-u) - e^{-iux}\phi(u)}{iu}\,du \quad (14)$$

Empirical performance of GH distributions in modeling skewness, kurtosis, and implied volatility smile in option prices made them more appealing than classical diffusion or jump-diffusion models. The hyperbolic law was found to provide a very good model for distributions of daily stock returns for a number of leading German enterprises (Eberlein and Keller, 1995), giving way to its use today in stock price modeling (Bibby and Sørensen, 1997) and market risk measurement (Eberlein et al., 1998). Eberlein et al. (1998) showed that the hyperbolic distribution allows an almost perfect fit to financial data, in both spot and derivatives markets.

---

[10] Specifically, $K_1(x) = \frac{x}{4}\int_0^\infty \exp\left[t + \frac{x^2}{4t}\right] t^{-2}\,dt$, $x \in R$.

V. Estimation of Oil Price Process as a Normal Inverse Gaussian Process

Estimation of the NIG distribution used the R program and the statistical packages: HyperbolicDist, ghyp, and fBasics package.[11, 12]

Table 2: Oil Price as Normal Inverse Distribution, Parameterization $(\alpha, \beta, \delta, \mu)$

	Parameters				Moments			
	Alpha	Beta	Delta	Mu	Mean	Variance	Skewness	Kurtosis
2000M1–2003M4	0.54	-0.02	2.69	0.08	-0.005	4.92	-0.08	1.72
2003M5–2007M10	0.97	-0.06	2.76	0.29	0.12	2.86	-0.11	1.13

Table 3: Oil Price as Normal Inverse Distribution, Parameterization $(\lambda, \bar{\alpha}, \mu, \sigma, \beta)$

	Parameters					Moments			
	Lambda	Alpha.bar Shape parameter	Mu Location parameter	Sigma Dispersion parameter	Beta Skewness parameter	Mean	Variance	Skewness	Kurtosis
2000M1–2003M4	-0.5	1.46	0.08	2.22	-0.08	-0.005	4.95	-0.08	2.06
2003M5–2007M10	-0.5	2.68	0.29	1.69	-0.17	0.12	2.87	-0.04	1.22

Estimation of the oil price NIG distribution yielded results consistent with findings for empirical distributions in Section II; namely, NIG parameters changed significantly in 2003M5–2007M10 compared with 2000M1–2003M4 under strong impulses from monetary policy. Location parameter μ increased from 0.08 to 0.29. Consequently, mean return increased from -0.005 (annualized to-0.005*255= -1.28 percent) in 2000M1–2003M4 to 0.12 (annualized to 30.6 percent) in 2003M5–2007M10. Scale parameter δ increased slightly from 2.69 to 2.76; however, by exceeding unity, it remained high, indicating a stretched out distribution. Shape parameter measuring skewness β remained small in the range of -0.02–-0.06, indicating symmetric oil price returns distribution. Shape parameter measuring tail steepness α increased significantly from 0.54 in 2000M1–2003M4 to 0.97 in 2003M5–2007M10, indicating steeper tails and therefore higher frequency of smaller jumps. Consequently, volatility fell considerably, from 2.2 in 2000M1–2003M4 (annualized to 35 percent) to 1.7 in 2003M5–2007M10 (annualized to 27 percent). In spite of this decline, volatility remained high for daily data,

---

[11] http://www.r-project.org/; The HyperbolicDist Package, David Scott d.scott@auckland.ac.nz; *The Ghyp Package*, Wolfgang Breymann, David Luethi, david.luethi@zhwin.ch; *The fBasics Package*, Diethelm Wuertz, wuertz@itp.phys.ethz.ch.

[12] Because characteristic function of NIG is known in closed form, parameter estimation can also be performed using the empirical characteristic function method (See Parzen 1962, Feuerverger and McDunnough, 1981).

implying that oil markets were constantly facing significant uncertainty and were sensitive to news and small shocks and prone to speculation.

In sum, parameter estimates for the two sub-periods were fully concordant with data. They established that a jump process with distinct features dominated the oil price process. In 2000M1–2003M4, oil prices exhibited high volatility, however, distribution mean was small and negative, indicating that the oil process fluctuated widely around a slightly declining trend. In 2003M5–2007M10, oil price process showed declining volatility; however, it was driven by a sharply upward trend, annualized to 30.6 percent per year, meaning that oil markets were permanently out-of-equilibrium during this period. While density was symmetric in both sub-periods, meaning that probability of upward jumps was matched by probability of downward jumps, the drift component of the process became powerful in the second sub-period and drove oil prices on a rising trend.

These parameters estimates can be explained by world demand and supply for crude oil and underlying fundamentals of crude oil markets. As world real GDP expanded at 4–5.5 percent per year during 2003–07 and the US dollar kept depreciating, world oil demand for oil expanded faster than before. Given rigidities in world oil supply, faster growth of oil demand created excess demand for oil. Given short-term inelasticity of oil demand and supply with respect to prices, any small excess demand for oil would cause large variation in prices. In turn, large price increases would have small negative effect on oil demand. A negative price effect, however, would be quickly dominated by positive income effect, i.e., world economic growth, which kept oil prices under rising pressure.

VI. Market Incompleteness and Esscher Transform

In this section we analyze the risk-neutral distribution, or equivalently, martingale measure,[13] associated with statistical NIG. Such a martingale measure is used for pricing derivatives based on NIG process. Indeed, except for the Brownian motion or the Poisson process, Levy Processes are incomplete models. A perfect hedge cannot be obtained and there is always a residual risk that cannot be hedged. In a Levy market, there are many different equivalent martingale measures under which discounted asset price process is a martingale. Existence of martingale measure is related to absence of arbitrage; while uniqueness of martingale measure is related to market completeness, i.e., perfect hedging.

[13] A discrete-time martingale is a discrete-time stochastic process $X_1, X_2, X_3, \ldots..$ that satisfies for all n: $E\left(\left|X_n\right|\right) < \infty$ and $E\left(X_{n+1} \mid X_1, \ldots.., X_n\right) = X_n$, i.e., conditional expected value of the next observation, given all of the past observations, is equal to the last observation.

One approach for finding an equivalent martingale measure is the Esscher transform proposed by Gerber and Shiu (1994). Given a statistical distribution P, the Esscher transform induces an equivalent probability measure Q and a martingale process. The Esscher parameter is determined so that discounted asset price is a martingale under the new probability measure Q. Let $S(t) = S(0)e^{X(t)}$, where $\{X(t)\}_{t\geq 0}$ is an LP with stationary and independent increments and $X(0) = 0$. For each t, the random variable $X(t)$, seen as the continuous compounded rate of return over t periods, has an infinitely divisible distribution with a probability density given by $f(x,t)$, $t > 0$. MGF, assumed to exist, is defined as $M(u,t) = E[e^{uX(t)}] = \int_{-\infty}^{\infty} e^{ux} f(x,t)dx$. Assuming $M(u,t)$ continuous at $t = 0$, it follows from infinite divisibility that $M(u,t) = [M(u,1)]^t$. Let h be a real number such that $M(h) = \int_{-\infty}^{\infty} e^{hx} f(x)dx$ exists; the Esscher transform (with parameter h) of $\{X(t)\}_{t\geq 0}$ is defined as an LP with stationary and independent increments, where the new probability density of $X(t)$, $t > 0$, is:

$$f(x,t;h) = \frac{e^{hx} f(x,t)}{\int_{-\infty}^{\infty} e^{hy} f(y,t)dy} = \frac{e^{hx} f(x,t)}{M(h,t)} \qquad (15)$$

The corresponding MGF is: $M(u,t;h) = \int_{-\infty}^{\infty} e^{ux} f(x,t;h)dx = \frac{M(u+h,t)}{M(h,t)}$ and $M(u,t;h) = [M(u,1;h)]^t$. An Esscher equivalent measure is given by:

$$\frac{dQ}{dP} = \frac{e^{hX_t}}{E(e^{hX_t})} = \exp\left(hX_t - t\log(M(h))\right) \qquad (16)$$

Accordingly, Esscher transform of NIG process has a MGF at $t = 1$ given by:

$$M(u,1;h) = \frac{M(u+h,1)}{M(h,1)} = e^{\mu(u+h)+\delta\left(\sqrt{\alpha^2-\beta^2}-\sqrt{\alpha^2-(\beta+u+h)^2}\right)-\left(\mu h+\delta\left(\sqrt{\alpha^2-\beta^2}-\sqrt{\alpha^2-(\beta+h)^2}\right)\right)}$$

$$= e^{\mu u+\delta\left(\sqrt{\alpha^2-(\beta+h)^2}-\sqrt{\alpha^2-(\beta+u+h)^2}\right)} \qquad (17)$$

The parameter h is determined so that the modified probability measure Q is an equivalent martingale measure to statistical probability measure P. The idea is to find $h = h^*$, so that the discounted stock price process $\{e^{-rt}S(t)\}_{t\geq 0}$ is a martingale with respect to the probability measure corresponding to h^*. The martingale condition is: $S(0) = E^Q[e^{-rt}S(t)] = e^{-rt}E^Q[S(t)]$. The parameter h^* is a solution to:

$$S(0) = E^Q[e^{-rt}S(t)] = e^{-rt}E^Q[S(0)e^{X(t)}] = e^{-rt}S(0)\frac{E^P[e^{(h+1)X(t)}]}{E^P[e^{hX(t)}]} = e^{-rt}S(0)\frac{M(1+h,t)}{M(h,t)} \qquad (18)$$

This condition is equivalent to the following equation: $1 = e^{-rt}E^{Q}[e^{X(t)}]$, or $e^{rt} = M(1,t;h^{*})$. The solution does not depend on t. Therefore setting $t=1$ yields $e^{r} = M(1,1;h^{*})$; in logarithm form, the parameter h is a solution to:

$$r = \log[M(1,1;h^{*})] = \log[M(1+h^{*},1)] - \log[M(h^{*},1)] \quad (19)$$

Applying the Esscher transform to NIG, and using MGF given by $M_{NIG}(u)$, the parameter h satisfies:

$$r = \mu(h+1) + \delta\left(\sqrt{\alpha^2-\beta^2} - \sqrt{\alpha^2-(\beta+(h+1))^2}\right) - \left(\mu h + \delta\left(\sqrt{\alpha^2-\beta^2} - \sqrt{\alpha^2-(\beta+h)^2}\right)\right)$$

$$r = \mu + \delta\left(\sqrt{\alpha^2-(\beta+h)^2} - \sqrt{\alpha^2-(\beta+h+1)^2}\right)$$

For parameters in Table 2, the parameter h was computed as h= -0.4858 for 2000M1–2003M4, and h=-0.5152 for 2003M5–2007M10. Esscher transforms would be:
$\frac{dQ}{dP} = \exp\left(-0.4858X_t + 0.2439t\right)$ and $\frac{dQ}{dP} = \exp\left(-0.5152X_t + 0.2691t\right)$, respectively.

An alternative approach for computing a risk-neutral measure, similar to the Esscher transform, can also be proposed (Carr et al., 2003). Let $(X_t)_{t\geq 0}$ be a real-valued process with independent increments, then $\left(\frac{e^{iuX_t}}{E[e^{iuX_t}]}\right)_{t\geq 0}$ is a martingale $\forall u \in R$. For example, if asset price S_t is modeled as $S_t = S_0 \exp\left(X_t\right)$ where X_t is an LP,[14] the resulting risk-neutral process for log-price is:

$$\log S(t) = (\log S(0) + rt - t\log E[\exp(X(t)]) + X(t)) \quad (20)$$

For NIG process, the risk-neutral process becomes:

$$\log S(t) = (\log S(0) + rt - \mu t - \delta t\left(\sqrt{\alpha^2-\beta^2} - \sqrt{\alpha^2-(\beta+1)^2}\right) + X(t)) \quad (21)$$

with $|\beta+1| < \alpha$. Characteristic function (CF) of log-price is:

$$E[\exp(iu\log(S(t)))] = \exp(\log S(0) + rt - \log E[\exp(X(t)])E[\exp(iuX(t))] \quad (22)$$

---

[14] In Madan et al. (1998), risk-neutral price process is $S_t = S_0\exp[rt + X_t + \omega t]$. Value for ω is determined by evaluating the CF for $X(t)$ at $u = \frac{1}{i}$, so that $e^{-rt}E\left(S_t\right) = S_0$; or equivalently $E\left(e^{X_t}\right) = e^{-\omega t}$. For NIG, $\omega = -\mu - \delta\left(\sqrt{\alpha^2-\beta^2} - \sqrt{\alpha^2-(\beta+1)^2}\right)$.

For NIG, risk-neutral CF is:

$$\phi_t\left(u\right)=\exp(\log S\,(0)+rt-\;\mu t\;-\delta t\left(\sqrt{\alpha^2-\beta^2}\;-\;\sqrt{\alpha^2-(\;\beta+1)^2}\;\right)\left(e^{\mu iu}\;\frac{e^{\delta\sqrt{\alpha^2-\beta^2}}}{e^{\delta\sqrt{\alpha^2-(\beta+iu\,)^2}}}\right)^t$$

$$=\exp(\log S(0)+rt-\;\mu t+\mu tiu+\delta t\left(\;\sqrt{\alpha^2-(\;\beta+1\,)^2}\;-\sqrt{\alpha^2-(\beta+iu\,)^2}\right))\quad(23)$$

As an illustration using the parameters in Table 2, and market data on November 2, 2007, namely futures oil price for end-December 2007 at US$95.93/barrel, US three-month treasury bill at r=4.595 percent, and taking t=57/365=0.16, risk-neutral CF for $\alpha=0.97$, $\beta=-0.06$, $\delta=2.76$, and $\mu=0.29$ would be:[15]

$$CF_{NIG}\;=7.47*\exp(0.046iu\;-0.442\sqrt{0.941-(-0.05+iu\,)^2}\,)$$

VII. Density Forecast of Crude Oil Prices: The Inverse Problem

In Section II we estimated the oil price process based on time-series data for oil price returns. Risk-neutral distribution was obtained by operating a transformation of the statistical distribution using many alternative techniques that have close resemblance to Girsanov's theorem (Duffie, 2001). In this section, risk-neutral distribution is derived from option prices in order to gauge market sentiment regarding future oil prices. This is the inverse problem in option pricing, consisting of estimating parameters of risk-neutral density from option prices. Inversion of option prices provides a density forecast for oil prices at a given maturity date T. In such forecast, besides expected mean, which is directly observed from futures prices, traders are also interested in volatility, skewness (direction of trends), and kurtosis (risk for large fluctuations).

Assuming a NIG distribution for log-price, the inverse problem can be stated as finding parameters $\theta=\left(\alpha,\beta,\delta,\mu\right)$ satisfying $\delta\geq 0$, $\mu\in R\,,0\leq\left|\beta\right|\leq\alpha$, by minimizing the quadratic pricing error:

$$\hat{\theta}=\arg\min_{\theta}\frac{1}{M}\sum_{j=1}^{M}\left(C_j^*(T,K_j)-C_j(T,K_j)\right)^2,\;\; j\;=1,2,....,M\quad(24)$$

Subject to put-call parity constraint:

$$S_0+P_j\left(T,K_j\right)-C_j^*\left(T,K_j\right)=K_je^{-rT}\qquad(25)$$

where $C_j^*(T,K_j)$ denotes call option price computed from NIG distribution, $C_j\left(T,K_j\right)$ and $P_j\left(T,K_j\right)$ denote, respectively, market call and put prices for maturity T and strikes K_j, S_0 is

[15] Parameters for 2000M1–2003M4 did not satisfy condition $\left|\beta+1\right|<\alpha$ and therefore would not yield real parameters for NIG distribution.

asset price at $t=0$, r is risk-free interest rate, and M denotes number of traded options (or strikes). Put-call parity condition brings extra-sample information that helps to regularize the estimation problem.[16] Choosing a penalty parameter $\ell \geq 0$, minimization problem becomes:

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{M} \sum_{j=1}^{M} \left(\left(C_j^*(T,K_j) - C_j(T,K_j) \right)^2 + \ell \left(S_0 + P_j\left(T,K_j\right) - C_j^*\left(T,K_j\right) - K_j e^{-rT} \right)^2 \right) \quad (26)$$

The above minimization requires knowledge of the functional form of $C_j^*\left(T,K_j\right)$. If the transition density of the process is known in closed form, then $C_j^*\left(T,K_j\right)$ can be derived as discounted expected payoff under a risk neutral density, namely:

$$C_j^*\left(T,K_j\right) = E^Q\left(\max\left(S_T - K_j, 0\right)\right) \quad (27)$$

However, noting that many LP may not have a density function that has a closed form, or have a density function that is not easily tractable, Heston (1993), Scott (1997), and Carr and Madan (1999) suggested the use of methods based on CF of a stochastic process to price options. Assuming CF is known analytically, many techniques become available for pricing options in the Fourier space. Let $s_t = \ln S_t$ be log-price, $k = \ln(K)$ log-strike price, and $p(s_T \mid s_t)$ risk-neutral density, then CF of $s_t = \ln S_t$ under risk-neutral measure is given by:

$$\phi_T(u) = \int_{-\infty}^{\infty} e^{ius_T} p(s_T \mid s_t) ds_T \quad (28)$$

Carr and Madan (1999) proposed Fast Fourier transform (FFT) method to compute option prices; they showed that option price can be written as:

$$C_j^*\left(T,K_j\right) = \frac{\exp(-ak_j)}{\pi} \int_0^{\infty} \operatorname{Re}\left[e^{-iuk_j} \psi_T(u)\right] du \quad (29)$$

Where $k_j = \ln K_j$, and $\psi_T(u)$ is Fourier transform of a modified call option price. In fact, defining the modified call option as $c_T(k) \equiv \exp(ak) C_T(k)$ for $a > 0$, its Fourier transform can be written as $\psi_T(u) = \int_{-\infty}^{\infty} e^{iuk} c_T(k) dk$. Carr and Madan (1999) showed that $\psi_T(u)$ can be expressed in terms of $\phi_T\left(u\right)$ as:

$$\psi_T(u) = \frac{e^{-rT} \phi_T(u - (a+1)i)}{a^2 + a - u^2 + i(2a+1)u} \quad (30)$$

[16] Cont and Tankov (2004) argued that inverse problem is an ill-posed problem and proposed relative entropy, which is the Kullback-Leibler distance for measuring proximity of two equivalent probability measures, as a regularization method with prior distribution estimated from statistical data via maximum likelihood method. This regularization enables the finding of a unique martingale measure.

For $a > 0$, singularity at $u = 0$ disappears. The option price can therefore be computed by FFT provided $\phi_T(-(a+1)i)$ is finite. In order to be able to use MGF, variable u is replaced by $-iu$.

The equivalent expression is: $\psi_T^{mgf}(u) = \dfrac{e^{-rT}\phi_T^{mgf}(u+(a+1))}{a^2+a+u^2+(2a+1)u}$

Where: $\phi_t^{mgf}(u) = e^{\mu u + \delta\left(\sqrt{\alpha^2-\beta^2}-\sqrt{\alpha^2-(\beta+u)^2}\right)}$ and $\alpha, \beta, \delta, \mu$ are here risk-neutral parameters.

Estimation of the implied risk-neutral distribution from option prices is a deconvolution problem. Madan et al. (1998) applied the maximum likelihood method to density function to calibrate a Variance Gamma process based on option prices. In this section, deconvolution methods based on CF are applied as CF necessarily satisfies the same differential equations or least squares problems as corresponding option prices. The estimation relies principally on the empirical characteristic function (ECF) method. The least squares are restated in Fourier space as:

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{N}\sum_{i=1}^{N}\left(\begin{array}{l}\left(\psi_T^{mgf}(u_i) - \dfrac{1}{M}\displaystyle\sum_{j=1}^{M} e^{u_i k_j} e^{ak_j} C_j(T,K_j)\right)^2 \\ +\ell\left(S_0\dfrac{1}{M}\displaystyle\sum_{j=1}^{M} e^{u_i k_j} e^{ak_j} + \dfrac{1}{M}\displaystyle\sum_{j=1}^{M} e^{u_i k_j} e^{ak_j} P_j\left(T,K_j\right) - \psi_T^{mgf}(u_i) - \dfrac{e^{-rT}}{M}\displaystyle\sum_{j=1}^{M} e^{u_i k_j} e^{ak_j} K_j\right)^2\end{array}\right) \quad (31)$$

$i = 1, 2, ..., N$, where N is the size of grid in Fourier space. Expression for $\psi_T^{mgf}(u)$ was given above.

To assert robustness of estimated parameters, an alternative calibration method was applied. Let $V = (C_1, ..., C_M, P_1, ..., P_M)'$ be a $(2M,1)$ vector of market call and put option prices, let D be a payoff matrix with dimensions $(2M, N)$ where $N \geq M$ is number of states at maturity T; V is related to empirical risk-neutral distribution q as follows:[17]

$$V = e^{-rT}.D.q \quad (32)$$

Risk-neutral distribution is computed using Tikhonov regularization method described in Engle et al. (1996) as:[18]

[17] This equation can be restated with a view to using the call-put parity condition. Let $D_C\ (M,N)$ be the payoff matrices associated with call options; let also, $V_C = (C_1, ..., C_M)'$ and $V_P = (P_1, ..., P_M)'$ be observed call and put option prices, V_K be a vector of strikes, and $V_1 = (1, ..., 1)'$ be the unit vector, then: $V_C = e^{-rT}.D_C.q$ subject to: $S_0 V_1 + V_P - e^{-rT}.D_C.q = e^{-rT}V_K$.

[18] Computation of $\hat{q}$ was carried out using the Matlab package by C. Hansen (1998): Regularization Tools A Matlab Package for Analysis and Solution of Discrete Ill-Posed Problems.

$$\hat{q} = e^{rT}.\left(D'D + \kappa.I\right)^{-1}.D'.V \qquad (33)$$

Where $\kappa > 0$ is a penalty parameter. Knowledge of $\hat{q}$ allows the estimation of parameters using ECF method. Define various states at time T as S_T^j , $j = 1, 2, ..., N$; each state is related to log-price returns X_T^j by $S_T^j = F_0^T e^{X_T^j}$,[19] where F_0^T is futures price at $t = 0$ for delivery at T . Define ECF as:

$$\phi_n(u) = \sum_{j=1}^{N} \exp(iuX_T^j)\hat{q}_j = \sum_{j=1}^{N} \cos(uX_T^j)\hat{q}_j + i\sum_{j=1}^{N} \sin(uX_T^j)\hat{q}_j \qquad (34)$$

and theoretical CF $\phi_{nig}(u)$ as:

$$\phi_{nig}\left(u\right) = \left(e^{\mu iu} \; \frac{e^{\delta\sqrt{\alpha^2-\beta^2}}}{e^{\delta\sqrt{\alpha^2-(\beta+iu)^2}}} \right) \qquad (35)$$

ECF is accordingly stated as:

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{N} \sum_{i=1}^{N} (\phi_n(u_i) - \phi_{nig}(u_i))' W (\phi_n(u_i) - \phi_{nig}(u_i)) , \; i = 1, 2, ..., N \qquad (36)$$

Where W is a positive semi-definite weighting matrix.

Knowledge of $\hat{q}$ also allows us to estimate parameters $\alpha, \beta, \delta, \mu$ using method of moments or, equivalently, method of cumulants. By computing empirical moments for a sample of state log-returns at maturity T based on empirical density $\hat{q}$, parameters $\alpha, \beta, \delta, \mu$ can be solved by equating sample moments with NIG theoretical moments given above in equations (9)–(12).

The market data used was for November 2, 2007; it consisted of call and put futures options contracts maturing end-December 2007; risk-free interest rate, taken here to be the three-month US Treasury bill rate, was equal to 4.595 percent; and crude futures price, was equal to US$95.93/barrel. Two methods for implying risk-neutral distribution were implemented. The first method computed Fourier transforms of call and put prices and applied constrained minimization as stated in (31). It yielded the following parameters: $\hat{\alpha} = 3.30$, $\hat{\beta} = 0.35$, $\hat{\delta} = 5.09$, and $\hat{\mu} = 1.75$. The second method used ECF as stated in (36) and applied General

---

[19] Probability density for X_T^j is the same as probability density for S_T^j. Indeed, if x is a random variable with probability density $f\left(x\right)$, for a monotone change of variable $y = g\left(x\right)$, the probability density of y , denoted $h\left(y\right)$, is given by: $h\left(y\right) = \left|\frac{1}{g'(g^{-1}\left(y\right))}\right| f\left(g^{-1}\left(y\right)\right)$ where g^{-1} is inverse function of g and g' is derivative. For $S_T = F_0 e^{X_T}$, the factor $\left|\frac{1}{g'(g^{-1}\left(y\right))}\right|$ is equal to 1.

Method of Moments to imply risk-neutral distribution: $\hat{\alpha} = 3.1$, $\hat{\beta} = 0.30$, $\hat{\delta} = 5.41$, and $\hat{\mu} = 1.74$. Applying equations (9-12) for NIG moments, the first method gave for expected mean 2.29 percent, variance 1.57 (sigma=1.25),[20] skewness 0.08, and kurtosis 0.19. For the second method, expected mean was 2.27 percent, variance 1.77 (sigma=1.33), skewness 0.07, and kurtosis 0.19. Therefore, market participants anticipated an average price for oil price of US$98/barrel= (95.93*exp(0.022)) for end-December 2007; dispersion measured by sigma was lower than statistical values in Tables 2 and 3, implying narrower interval of variation around expected mean; skewness was positive implying higher probability for oil prices to rise above expected mean than to fall below this mean. Kurtosis was below 3, implying flatter NIG distribution compared with normal distribution and higher than normal probability for tail events.

Based on implied crude oil density forecast, market participants' short-term expectations seemed to be strongly influenced by underlying fundamentals characterizing oil markets. These fundamentals were characterized by expansionary monetary policy since 2001, sharply depreciating US dollar, which had fallen by over 65 percent vis-à-vis Euro since 2001, higher world economic growth and consequently higher demand for oil. Given crude oil supply rigidities, traders expected excess demand for crude oil to increase, and consequently to cause further pressure on oil prices. Moreover, traders were cognizant that oil markets were not singularly and separately affected by monetary shocks; other commodities markets were experiencing similar shocks. World aggregate demand for commodities had expanded, resulting in double digit inflation for commodities prices, estimated at about 23 percent per year in 2003M5–2007M10. Such high inflation would contribute to rapidly erode real interest rates, and therefore to stimulate further real aggregate demand for goods and services. Financial conditions, characterized by sub-prime markets defaults, large write-offs by leading banks, and piling up of credits, showed that restraints in monetary policy were not in the offing soon. This was demonstrated by further relaxation of monetary policy in August-December 2007, followed by immediate weakening in US dollar and surge in oil prices. All this surrounding information might have led traders to assume persistence in oil prices and indicating increased likelihood of right tail events.

VIII. CONCLUSIONS

Oil prices have been relentlessly on a rising path under strong impetus from faster growing oil demand and lagging oil supply. Using NIG distribution, a subclass of GH distribution which have been found to closely fit high frequency financial data, we obtained the result that oil price parameters changed dramatically in 2003M5-2007M10 in comparison to 2000M1–2003M4.

[20] It is known that change of measure from statistical to risk-neutral distribution does not change variance; it changes only expected returns. Computation not reported here showed that variance of oil returns dropped sharply during July-October, 2007.

Changes were characterized by high mean return that rose from –0.005 to 0.11, and lower dispersion that fell from 2.22 to 1.70. NIG for both sub-periods had low kurtosis, implying flatter than normal distribution. Based on NIG parameters, oil prices would be expected to rise at about 30 percent per year.

Crude oil density forecast for the end of December 2007 was extracted from option prices data on November 2, 2007. Traders' expectations were in full accordance with prevailing market fundamentals, characterized by widening oil demand-supply gap, the highest post WWII commodity price inflation, falling real interest rates, a depreciating US dollar, and a meltdown of the sub-prime debt market. Relaxation of monetary policy in August─December 2007 jolted oil prices and apparently contributed to firming up of market expectations toward rising oil prices. Derived risk-neutral density had low kurtosis, implying flatter distribution and significant probability for tail events.

A legitimate question is how far oil prices could rise without reaching critical zone of triggering a world recession, or unexpectedly, a drop in oil supply? Or equivalently, as oil and other commodities markets were strongly affected by monetary shocks, how far monetary stance can remain accommodative without exacerbating inflation and causing recession? The answer is most likely not too far, noting oil price rising trends were simultaneously accompanied by fast rising trends in other commodities prices, fast depreciating currencies, weakening financial conditions, and write-offs on bad debt. Persistence of present trends would culminate in explosive commodities prices and could turn out to be un-sustainable. More worrisome, oil producers have become wary of rapidly falling value of their international reserves, which could discourage oil supply.

Although it could be claimed that high commodities prices had not so far dented world economic growth and had not affected consumer price indices, recessionary and inflationary implication of oil price shocks should not be underestimated. Hamilton (1983) and Hamilton and Herrera (2004) have analyzed recessionary effects of oil prices and allowed for longer lag for these effects to be fully transmitted to output and prices. The relationship between oil prices and GDP and inflation has also been studied by Bernanke et al. (1997), Jones et al. (2004), and Lee et al. (1995). These authors found significant recessionary and inflationary impact for oil prices on real GDP and consumer prices.

We have shown that in 2000M1–2003M4 oil prices were evolving around slightly declining trend with moderate oil demand growth. Such scenario could be restored provided restrained monetary policy is in the works. However, a policy dilemma would face policymakers as lessons from past inflationary experiences in many countries had clearly demonstrated: restraining monetary policy with attendant temporary recession, or risking high inflation with attendant recession, financial disorder, and social unrest. High inflation may discourage supply of goods in general, as value of money is falling precipitately. If oil supply turns regressive, economic growth would be impeded, and pressure on oil prices will accelerate.

Restoring stability to oil markets is essential for durable economic growth and price stability. The brunt of this burden is evidently on monetary policy. The latter cannot be used as a panacea for all. Different types of economic issues may be best addressed through well-targeted instruments and appropriate solutions. For instance, balance of payments deficits could be best and quickly achieved via monetary approach to balance of payments which consists of reducing public and private deficit financing through credit ceilings. Similarly, external competitiveness could be durably achieved via productivity gains, cost reduction, and technical innovations without necessarily trying to depress nominal exchange rates. Exchange rate depreciation may not restore external competitiveness in context of expansionary monetary policy. To be effective, exchange rate has to be supported by restrictive monetary policy. Safe conduct of monetary policy is a prerequisite for economic stability and growth.

In sum, oil prices are amongst key economic variables. By juxtaposing two sub-periods, we have shown that oil price parameters could be sensitive to macroeconomic policies. Accordingly, prudent monetary policy may be necessary for achieving longer-term oil price stability and growth.

REFERENCES

Barndorff-Nielsen, O.E, 1977, “Exponentially Decreasing Distributions for the Logarithm of Particle Size,” *Proceedings of the Royal Society London A* 353: pp. 401–419.

Barndorff-Nielsen, O.E. and Blaesild, P., 1983, “Hyperbolic Distributions.” In Encyclopedia of Statistical Sciences, eds., Johnson, N. L., Kotz, S. and Read, C. B., Vol. 3, pp. 700–707. New York: Wiley.

Barndorff-Nielsen, O.E., 1995, “Normal Inverse Gaussian Processes and the Modeling of Stock Returns”, *Research Report* 300, Department Theoretical Statistics, Aarhus University.

Barndorff-Nielsen O.E., 1997, “Processes of Normal Inverse Gaussian Type," *Finance and Stochastics*, 2, pp. 41–68.

Bernanke, B.S., M. Gertler and M. Watson, 1997, “Systematic Monetary Policy and the Effects of Oil Shocks,” *Brookings Papers on Economic Activity*, Vol. 1997, No. 1, pp. 91–157.

Bibby B M and Sorensen M, 1997, A Hyperbolic Diffusion Model for Stock Prices, *Finance and Stochastics*, 1, pp. 25–41.

Bibby, B. M. and Sørensen, M., 2003, “Hyperbolic Processes in Finance.” In Handbook of Heavy-Tailed Distributions in Finance, ed., Rachev, S. T. pp. 212–248. Elsevier Science, B.V.

Black, F., and M. Scholes, 1973, “The pricing of Options and Corporate Liabilities,” *Journal of Political Economy,* Vol. 72, pp. 637–659.

Breymann, Wolfgang and David Luethi, 2007, *The Ghyp Package*.

Carr, P., H., Geman, D. Madan, and M. Yor, 2003, “Stochastic Volatility for Levy Processes,” *Mathematical Finance*, Vol. 13, No. 3, pp. 345–382.

Carr, P., and D. Madan, 1999, “Option Valuation Using Fast Fourier Transform,” *Journal of Computational Finance,* Vol. 2, pp. 61–73.

Clark, P. K., 1973, “A Subordinated Stochastic Process with Finite Variance for Speculative Prices,” *Econometrica,* Vol. 41, pp. 135-155.

Cont, R. and Tankov, P., 2004, *Financial Modeling with Jump Processes*, (Chapman&Hall/CRC).

Dagpunar, J.S., 1989, "An easily Implemented Generalized Inverse Gaussian Generator," *Commun.Statist.Simula*., 18, pp. 703–710.

Davies, R. B., 1973, "Numerical Inversion of a Characteristic Function," *Biometrika*, 60, pp. 415-417.

Duffie, D., 2001, *Dynamic Asset Pricing Theory*, Third English Edition, Princeton University Press.

Eberlein, E. and Keller, U., 1995, "Hyperbolic Distributions in Finance," *Bernoulli* 1: pp. 281-299.

Eberlein, E., U. Keller, and K. Prause, 1998, "New Insights into the Smile, Mispricing and Value-at-Risk: The Hyperbolic Model," *Journal of Business,* Vol. 71, pp. 371–406.

Engle, H. W., Hanke, M., and Neubauer, A., 1996, *Regularization of Inverse Problems*, Kluwer, Dordrecht.

Fama, E.F., 1965, "The Behavior of Stock Market Prices," *Journal of Business,* Vol. 34, pp. 420-429.

Feuerverger, A. and P. McDunnough, 1981, "On Some Fourier Methods for Inference," *Journal of the American Statistical Association,* Vol. 78, No 375, pp. 379–387.

Gerber, H. U., and E. S. W. Shiu, 1994, "Option Pricing by Esscher Transforms," *Transactions of the Society of Actuaries,* Vol. 46, pp. 99–191.

Hamilton, J.D., 1983, "Oil and the Macroeconomy since World War II," *Journal of Political Economy*, 91, pp. 228–48.

Hamilton, J.D. and A.M. Herrera, 2004, "Oil Shocks and Aggregate Macroeconomic Behavior: The Role of the Monetary Policy," *Journal of Money, Credit, and Banking,* Vol. 36, No. 2, pp. 265–86.

Hansen, C., 1998, *Regularization Tools A Matlab Package for Analysis and Solution of Discrete Ill-Posed Problems*.

Heston, S. L., 1993, "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bonds and Currency Options", *The Review of Financial Studies*, Vol. 6, No. 2, pp. 327–343.

Jones, D.W., P.N. Leiby, and I.K. Paik, 2004, "Oil Price Shocks and the Macroeconomy: What Have We Learned Since 1996?" *The Energy Journal*, Vol. 25, No. 2, pp. 1–32.

Lee, K., S. Ni, and R.A. Ratti, 1995, "Oil Shocks and the Macroeconomy: The Role of Price Variability," *The Energy Journal,* Vol. 16, No. 4, pp. 39–56.

Madan, D. B., Carr, P. P. and Chang, E. C., 1998, "The Variance Gamma Process and Option Pricing", *European Finance Review* 2: pp. 79–105.

Mandelbrot, B., 1963, "New Methods in Statistical Economics," *Journal of Political Economy,* Vol. 61, pp. 421–440.

Parzen, E, 1962, "On Estimation of a Probability Density Function and Mode," *Ann. Math. Statist.,* Vol 33, pp. 1065–1076.

Prause, K., 1999, *The generalized hyperbolic models: Estimation, financial derivatives and risk measurement*. PhD Thesis, Mathematics Faculty, University of Freiburg.

Rydberg, T. H., 1997, "The normal inverse Gaussian Lévy process: Simulation and approximation," *Commun. Statist.–Stochastic Models*, Vol. 34, pp. 887–910.

Scott, D., 2007, *The HyperbolicDist Package*.

Scott, L. O., 1997, "Pricing Stock Options in a Jump-Diffusion Model with Stochastic Volatility and Interest Rates: Applications of Fourier Inversion Methods", *Mathematical Finance*, Vol. 7, No. 4, pp. 413-424.

Wuertz, Diethelm, 2007, *The fBasics Package*.